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Notre Dame, IN 46556. *Computability of integer parts.*

An *integer part* of a real closed field R is a discrete ordered subring I containing 1 such that for all $r \in R$ there exists a unique $i \in I$ with $i \leq r < i + 1$. Mourgues and Ressayre showed that every real closed field R has an integer part. For a countable real closed field R , we previously showed that the integer part obtained by the procedure of Mourgues and Ressayre is $\Delta_{\omega\omega}^0(R)$. We would like to know whether there exists a construction that yields a computationally simpler integer part, perhaps one that is $\Delta_2^0(R)$. All integer parts are *Z-rings*, discretely ordered rings that have the euclidean algorithm for dividing by integers. By a result of Wilkie, any *Z-ring* can be extended to an integer part for *some* real closed field. We show that we can compute a maximal *Z-ring* I for any real closed field R that is $\Delta_2^0(R)$, and we then examine whether this I must serve as an integer part for R . We also show that certain subclasses of $\Delta_2^0(R)$ are not sufficient to contain integer parts for a real closed field R . (Received February 10, 2011)