Joshua Holden (holden@rose-hulman.edu), Department of Mathematics, Rose-Hulman Insitute of Technology, Terre Haute, IN 47803, and Margaret M Robinson* (robinson@mtholyoke.edu), Department of Mathematics, 50 College Street, South Hadley, MA 01075. Counting fixed points, two-cycles, and collisions of the discrete logarithm using p-adic methods. Preliminary report.
Brizolis asked for which primes does there exist a pair $(g, h)$ such that $g^{h} \equiv h \bmod p$. To rephrase, he asked if for $p>3$ there is always a pair $(g, h)$ such that $h$ is a fixed point of the discrete $\operatorname{logarithm} \log _{g}$. Zhang (1995) and Cobeli and Zaharescu (1999) answered with a "yes" for sufficiently large primes and gave estimates for the number of such pairs when $g$ and $h$ are primitive roots modulo $p$. In 2000, Campbell showed that the answer to Brizolis was "yes" for all primes. The first author has extended this question to questions about counting fixed points, two-cycles, and collisions (pairs ( $h, a$ ) where $h^{h} \equiv a^{a} \bmod p$ where $h$ and $a$ are not necessarily primitive roots). In this paper, we use $p$-adic methods, primarily Hensel's lemma and $p$-adic interpolation, to count fixed points, two cycles, and collisions given certain conditions on $g$, $h$, and $a$ modulo powers of a prime $p$. (Received February 14, 2011)

