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Chris D. Lynd* (chris_lynd@my.uri.edu), chris_lynd@my.uri.edu, RI. *Using Difference Equations to Extend Our Knowledge of Nested Radicals.*

The sequence

$$\{z_n\}_{n=1}^{\infty} = \{c_0 \sqrt[r_1]{a_1}, c_0 \sqrt[r_1]{a_1 + c_1 \sqrt[r_2]{a_2}}, \dots, c_0 \sqrt[r_1]{a_1 + c_1 \sqrt[r_2]{a_2 + \dots + c_{n-1} \sqrt[r_n]{a_n}}, \dots\}$$

when defined, is denoted by the right nested root

$$c_0 \sqrt[r_1]{a_1 + c_1 \sqrt[r_2]{a_2 + c_2 \sqrt[r_3]{a_3 + c_3 \sqrt[r_4]{a_4 + \dots}}}}$$

We consider right nested roots where $\{c_n\}_{n=0}^{\infty}$ and $\{a_n\}_{n=1}^{\infty}$ are periodic sequences of real numbers and $\{r_n\}_{n=1}^{\infty}$ is a periodic sequence of integers greater than or equal to two. We show that right nested roots of this form can be produced from solutions to a first-order difference equation $x_{n+1} = f(x_n)$ for $n = 1, 2, 3, \dots$ where f is a continuous, monotone function.

We use the equilibrium points and periodic points of the difference equation, and their basins of attraction, to determine the convergence and limit points of the corresponding nested root. Our method of analysis differs from previous work and extends previous convergence results to r th roots, periodic parameters with an arbitrary period, and negative parameters. It also extends previous convergence results for left nested roots and can be applied to continued fractions with periodic parameters. (Received February 13, 2011)