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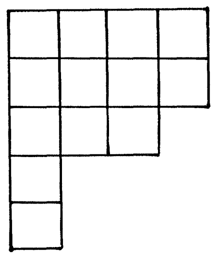
The collected papers of Alfred Young 1873–1940, G. de B. Robinson (editor), University of Toronto Press, Toronto and Buffalo, 1977, xxvii + 684 pp., $10.00.

The twenty seven papers of the Reverend Alfred Young are attractively collected in this volume together with a foreword by G. de B. Robinson and Young's obituary by H. W. Turnbull. The papers were all written over forty years ago (although one was published posthumously in 1952), and as Turnbull says in the obituary:

"Young's work is never easy reading, for it lacks that quality which helps the reader grasp the essential point at the right time. The very closest and constant attention is required to pick out some of the most fundamental results from a mass of detail. One could almost suppose that he camouflaged his principal theorems. His work resembles a noonday picture of a magnificent sunlit mountain scene rather than the same in high relief with all the light and shade of early morning or sunset."

It is natural then to ask whether it is worthwhile to publish a volume of old obscure papers. To answer this we shall examine some of the ramifications of Young's ideas in recent research. First of all two recent conferences *Combinatoire et representation du groupe symetrique* in Strasbourg [**19**] and *Alfred Young Day* in Waterloo [**82**] were both centered on the theme of Young's research.

Young's most important achievements lie in his series of nine papers *Quantitative substitutional analysis* (QSA) which occupy well over half of the present volume. In the first QSA paper Young introduces the method of tableaux. A *Young tableau* is an array of the first p integers (Young uses p letters) constructed as follows: Let π be any partition of the integer p into positive parts. The Ferrers graph of π (needed for the tableau construction) may best be understood by an example; if π is the partition $4 + 4 + 3 + 1 + 1$, the Ferrers graph of π is



The ith row of boxes corresponds to the ith part of π. In the Ferrers graph of π insert the integers $1,2, \ldots, p$; such an array constitutes one of the $p!$ possible Young tableaux of shape π. If in addition there is strict increase in each row and column, the array is called a standard Young tableau (introduced in QSA III). For example, if $p = 5$ and π is the partition $3 + 2$, then the five standard Young tableaux of shape π are

$$\begin{matrix} 1\,2\,3 & 1\,2\,4 & 1\,2\,5 & 1\,3\,4 & 1\,3\,5 \\ 4\,5 & 3\,5 & 3\,4 & 2\,5 & 2\,4 \end{matrix}.$$

Young's object in the entire QSA series was to treat problems in invariant theory; for example, he showed in QSA I that the method of tableaux could be used to replace the polarization operator in the derivation of the Clebsch-Gordon series.

Frobenius [**24**] (see also [**25**]) observed that the method of tableaux was closely connected with his own work on the representation theory of the symmetric group. Young felt compelled to master the contributions of Frobenius (and Schur [**102**]) before continuing his QSA series. However since Young was a country pastor (Rector of Birdbrook, Essex, 1910–1940) and no linguist, a period of twenty five years elapsed between QSA II and QSA III, a paper primarily devoted to the irreducible representations of the symmetric group. The keystone to the work in QSA III and QSA IV lies in the explicit construction from the Young tableaux of certain elements of the group algebra that Young called the positive and negative symmetric groups. From the totality of these elements of the group algebra corresponding to all tableaux of a fixed shape π, specific idempotents of the group algebra are constructed, and from these Young constructs the actual matrices of the irreducible representation of the symmetric group S_n corresponding to the partition π of n. A full account is given by Rutherford [**100**].

Young's ideas have had a significant impact on: (1) group representation theory, (2) combinatorics and statistics, (3) invariant theory, (4) physics and (5) chemistry. These topics intertwine sufficiently that a single result may have implications in more than one area.

Research on the representation theory of the symmetric group has advanced tremendously in recent years. While Young was concerned only with representations over a field of characteristic zero, much has since been done to treat the case of arbitrary characteristic (the "modular" representation theory). Among the most interesting work in this area is that of G. D. James. An exposition of his accomplishments and those of others on this topic occurs in **[51]**. Also an excellent treatment of related results concerning the Hall polynomials and symmetric functions has recently been given by I. G. Macdonald **[68]**.

The combinatorial aspects of Young's work often seem to arise from group theory. For example, the concept of "hook length" concerns a set of parameters related to Young tableaux. These arose first in the work of Nakayama **[77]**, **[78]** on modular representations of the symmetric group. However the idea of hook length has numerous further implications for group theory **[23]** and for combinatorics **[20]**, **[36]**, **[108]**.

Another combinatorial feature of Young tableaux is the Robinson-Schensted-Knuth correspondence. This is a subtle combinatorial algorithm for constructing bijections between certain sets of matrices and sets of "generalized" Young tableaux. For example it is possible to exhibit a one-to-one correspondence between the set of symmetric $p \times p$ permutaton matrices and Young tableaux with p parts using this correspondence. The algorithm arises in the work of Robinson **[88]**, **[89]**, **[90]**, and it was later rediscovered by Schensted **[101]**. Schensted's ideas were extended by Schützenberger **[104]**, **[105]**, and Knuth **[53]**. One can perhaps appreciate the combinatorial clout of this work by recalling a difficult result of Erdös and Szekeres **[15]**.

THEOREM. *Any permutation of the integers* $1, 2, 3, \ldots, n^2 + 1$ *contains either an increasing subsequence of length* $n + 1$ *or a decreasing subsequence of length* $n + 1$.

Schensted **[101]** using the algorithm just described greatly strengthened this result by proving the

THEOREM. *The number of permutations of* $1, 2, \ldots, m$ *with longest increasing subsequence of length* c *and longest decreasing subsequence of length* r *is equal to* $\Sigma_\mu (f_\mu)^2$, *where the sum is over all partitions* μ *of* m *with largest part equal to* r *and number of parts equal to* c. *The term* f_μ *is the number of standard Young tableaux of shape* μ.

Note that the Erdös-Szekeres theorem follows easily from Schensted's result since any partition of $n^2 + 1$ must either have more than n parts or have at least one part that exceeds n.

G. Kreweras (**[55]**, **[56]**, **[57]**, **[58]**, **[59]**, **[60]**, **[61]**) has obtained many fundamental results on Young tableaux by considering what he calls the *Young lattice*. This is the lattice of all partitions of integers ordered by $\lambda \geqslant \mu$ if and only if for each i the ith part of λ is at least as large as the ith part of μ.

The classical ballot problems which are related to numerous problems in statistics are also related to Young tableaux. Here one considers k candidates for office and n votes distributed among them. It is postulated that candidate 1 wins, candidate 2 comes in second, etc..

Problem. How many ways can the ballots be counted so that at each step of the counting no one has more votes than candidate 1, no one except possibly candidate 1 has more votes than candidate 2, etc.? For example, if amongst 5 votes, candidate 1 gets two votes, candidate 2 gets two votes and candidate 3 gets one vote, then the five admissable counting arrangements of the ballots are 11223, 11232, 12123, 12132 and 12312. Such "counting arrangements" are generally called "lattice permutations". These lattice permutations are in one-to-one correspondence with the standard Young tableaux of shape $2 + 2 + 1$; the corresponding Young tableau is constructed by putting i in the jth row if the ith entry of the lattice permutation is j. The ballot problem may now be persued with all the available results on Young tableaux. Barton and Mallows **[3]** in their paper on the random sequence give more details and describe related problems as does Stanley **[108]** (see also Nakayama **[78]**, Narayana **[79]**, **[80]**, MacMahon **[69]** and Steck **[109]**, **[110]**).

Concerning invariant theory, popular belief has it that Hilbert **[35]** killed the subject. An extensive sociological post-mortem was given by Fisher **[17]**. All this, of course, flies in the faces of the rejuvenation of invariant theory in theoretical physics, and Hermann Weyl's compelling case for its importance **[118]**. More recently a number of mathematicians have recognized the continuing significance of the subject. D. Mumford **[75]** utilized invariant theory in his solution of the problem of "moduli" of algebraic curves; Dieudonné and Carrell **[13]** provide a nice introduction to the work of Mumford. Rota et al. **[94]**, **[95]**, **[96]** consider the relationship between invariant theory and modern work in combinatorics; indeed, these papers may be viewed as direct modern outgrowths of Young's work.

Theoretical chemistry has also found invariant theory and Young's constructive approach to the representation theory of the symmetric group to be of value. V. Prelog **[84]** in his Nobel Lecture *Chirality in Chemistry* refers to Young's work. An object is "chiral" if it cannot be transformed into its mirror image through translation and rotation; this geometric aspect of certain molecules turns out to be of significance in chemistry. E. Ruch et al. **[63]**, **[97]**, **[98]**, **[99]** consider Young's representation theory of the symmetric group in order to achieve structural insight into chirality in chemistry.

In physics Young tableaux also arise. Extensive work by L. C. Biedenharn and J. D. Louck (see **[9]**) concerns the representations of the unitary group and the theory of bounded operators defined on the Hilbert space $H = \Sigma_{[m]} \oplus H^{[m]}$ where $H^{[m]}$ is the carrier space of a unitary irreducible representation of $U(n)$. It turns out that the basis vectors of $H^{[m]}$ are in one-to-one correspondence with the set of standard Young tableaux of shape $[m]$. Louck has pointed out to me that physicists have generally employed Gel'fand patterns instead of Young tableaux for denoting the basis vectors of $H^{[m]}$. Apparently Baird and Biedenharn **[1]** first pointed out the one-to-one correspondence between Gel'fand patterns and standard Young tableaux. These and other applications of Young tableaux in physics are touched on in **[9]**.

Thus there can be no doubt of the fruitfulness of Young's work. Surely the above incomplete survey indicates its extensive impact. The appearance of *Young's collected papers* should assist future mathematicians in the difficult but rewarding task of understanding Young's ideas. On this very point, we must mention the papers by Garsia and Remmel **[26]**, **[27]** who were led to a valuable study of Young's raising operator as a result of trying to reconcile an apparent contradiction in two of Young's formulas. In a similar vein, G. D. James wrote (in a letter to me) concerning the concealed gems in Young's work: "Murphy and I have recently proved Carter's Conjecture determining which ordinary irreducible representations (for p-regular diagrams) remain irreducible modulo p. After doing so, I noticed Theorem VI on page 460 of the Collected Works, and after two or three days discovering what was going on, I realized that this contains most of the crucial information. No doubt if Young had been presented Carter's Conjecture he would have proved it in a very short time . . . "

In our list of references we have tried to include a majority of the papers that have cited Young within the past 15 years. The following table lists the general area to which each of those papers belongs.

GROUP REPRESENTATION THEORY. **[4]**, **[5]**, **[6]**, **[7]**, **[23]**, **[26]**, **[27]**, **[32]**, **[33]**, **[34]**, **[40]**, **[41]**, **[42]**, **[43]**, **[44]**, **[45]**, **[46]**, **[47]**, **[48]**, **[49]**, **[50]**, **[51]**, **[52]**, **[54]**, **[64]**, **[65]**, **[66]**, **[67]**, **[68]**, **[70]**, **[71]**, **[72]**, **[73]**, **[74]**, **[76]**, **[77]**, **[78]**, **[85]**, **[87]**, **[88]**, **[89]**, **[90]**, **[91]**, **[100]**, **[102]**, **[104]**, **[105]**.

COMBINATORICS AND STATISTICS. **[2]**, **[3]**, **[8]**, **[10]**, **[12]**, **[14]**, **[18]**, **[19]**,**[20]**, **[21]**, **[22]**, **[29]**, **[30]**, **[31]**, **[36]**, **[37]**, **[38]**, **[39]**, **[53]**, **[55]**, **[56]**, **[57]**, **[58]**, **[59]**, **[60]**, **[61]**, **[79]**, **[80]**, **[81]**, **[101]**, **[106]**, **[107]**, **[108]**, **[109]**, **[110]**, **[111]**, **[112]**, **[113]**, **[117]**

INVARIANT THEORY. **[13]**, **[28]**, **[92]**, **[93]**, **[94]**, **[95]**, **[96]**, **[103]**, **[114]**, **[115]**, **[116]**, **[118]**, **[119]**, **[120]**

PHYSICS. **[1]**, **[9]**, **[11]**, **[16]**, **[75]**, **[121]**

CHEMISTRY. **[62]**, **[63]**, **[84]**, **[97]**, **[98]**, **[99]**

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Locally solid Riesz spaces, by Charalambos D. Aliprantis and Owen Burkinshaw, Academic Press, New York, 1978, xii + 198 pp.

Vector lattices, also called Riesz spaces, have been objects of mathematical interest at least since F. Riesz's pioneering paper [34] at the International Mathematical Congress held at Bologna in 1928. Since then many others have developed the subject. Some of the more important contributions to the theory through 1950 were made by the following authors. H. Freudenthal [14], S. W. P. Steen [37], L. V. Kantorovich [19], M. H. Stone [38], H. Nakano [26], [27], [28], [29], [30], [31], [32], F. Maeda and T. Ogasawara [25], [33], K. Yosida [40], [41], [42], H. F. Bohnenblust [9], S. Kakutani [17], [18].

In the next fifteen years vector lattices were not given much attention. Some important things were done. A paper of I. Amemiya [1] gave many new advances in the algebraic theory, some of which are still being rediscovered. W. A. J. Luxemburg and A. C. Zaanen were also very active at this time with a succession of important papers [22], [23].