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CORRIGENDUM
PARITY OF CLASS NUMBERS AND WITT EQUIVALENCE OF QUARTIC FIELDS
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The proof of the Proposition on p. 1712 incorrectly interprets the nondegeneracy of the Hilbert symbol. Here is the Proposition with corrected proof.

Proposition. *Let F be a number field such that $s(F) = 2$ and $s_1 = \cdots = s_g = 1$. Then the class number of F is even, and every number field Witt equivalent to F has also even class number.*

Proof. Take a nondyadic prime $\mathfrak{q}$ of F with $s(F_\mathfrak{q}) = 2$ (there must be one since $s(F) = 2$). We show that the order d of the ideal class $[\mathfrak{q}]$ in the ideal class group of F is even. For suppose d is odd. Then $\mathfrak{q}^d = (a)$ is a principal ideal and the number a is a $\mathfrak{q}$-adic prime times a $\mathfrak{q}$-adic square. Since -1 is not a local square at $\mathfrak{q}$, we have $(-1, a)_\mathfrak{q} = -1$. On the other hand, we claim that

$$(-1, a)_\mathfrak{p} = 1$$

for all the remaining primes $\mathfrak{p}$ of F. For a dyadic prime $\mathfrak{p}$ this is obvious since -1 is a local square at $\mathfrak{p}$ (the local dyadic levels are all equal to 1). When $\mathfrak{p}$ is a finite nondyadic prime and $\mathfrak{p} \neq \mathfrak{q}$, then a is a local unit at $\mathfrak{p}$ (by our choice of a), hence again the Hilbert symbol is trivial. Since $s(F) = 2$, there are no real infinite primes, and at complex infinite primes any Hilbert symbol is trivial. This proves our claim, contradicting Hilbert reciprocity law. Hence d must be even, and so also the ideal class number of F is even.

If K is any number field Witt equivalent to F, then K and F have the same Witt equivalence invariant. Hence, if F satisfies Conner's level conditions, so does K, and, as has been already proved, it has even class number. □

We take this opportunity to give more precise information about cubic number fields. We have mentioned in the paper that every cubic field is Witt equivalent to a field with odd class number. It is a fact that the representatives of the eight cubic Witt equivalence classes discussed in the paper all have class number one. Thus every cubic field is Witt equivalent to a field with class number one.

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