Four Reactions to *Principles and Standards for School Mathematics*

Susan Addington, Herbert Clemens, Roger Howe, and Mark Saul

In April 2000 the National Council of Teachers of Mathematics (NCTM) published *Principles and Standards for School Mathematics* (PSSM). This document is an updated and unified version of the school mathematics standards issued by the NCTM over several years starting in 1989. The *Notices* invited four individuals to write short pieces describing their reactions to *Principles and Standards*; these pieces appear below.

The *Notices* has published two other pieces about PSSM: a news item, "Updated NCTM Standards Released", June/July 2000, pages 683–684; and an article, "Principles and Standards for School Mathematics: A Guide for Mathematicians", by Joan Ferrini-Mundy, September 2000, pages 868–876.

—Allyn Jackson

Susan Addington

Do the updated NCTM standards provide a good blueprint for reforming K–12 mathematics education in the United States?

Yes. But reforming K–12 mathematics education in the U.S. is like turning the *Titanic* around with a canoe paddle or perhaps like herding a million cats.

Despite the lack of national directives, the U.S. K–12 educational system is amazingly uniform and deeply set in its ways. The Third International Mathematics and Science Study documented how mathematics teaching in the U.S. is overwhelmingly "a mile wide and an inch deep," covering dozens of topics superficially. In a 1999 book, Stigler and Hiebert[1] characterized U.S. mathematics teaching as "learning terms and practicing procedures." I have found this to be accurate in most of the classrooms I have visited, despite some notable exceptions.

On the other hand, the long history in the United States of local control of schools and Americans' general distrust of directives from the federal government mean that every state has its own standards and every school district chooses its own curricula (sometimes several).

The U.S. has some current urgent problems. Politicians, noticing that citizens now care about education, have instituted quick fixes such as high-stakes standardized tests, with budgets cut, principals fired, and students failed if they do not measure up. Most large states are suffering dire teacher shortages, especially in urban schools; mathematics is one of the subjects suffering the most. Most of the population thinks that learning mathematics *is* learning terms and practicing procedures, as do most elementary school teachers and some middle and high school teachers. Even if a consensus were reached about what to teach and how, it takes time for teachers to learn new material and adjust to new teaching methods. Three times through a lesson is a good rule of thumb for mastering a new teaching technique; for most K–12 teachers this means three years. And districts, for the most part, have not accepted the fact that teachers need time every day to discuss and reflect on teaching and mathematics with their colleagues.

So how could a bunch of mathematics teachers get all these cats on the same track and steer the oceanliner of U.S. education away from the icebergs?

In fact, the 1989 NCTM *Standards* were the first such set of subject-matter standards developed by a professional organization; standards in other subjects soon followed. The NCTM standards were

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[1] *James W. Stigler and James Hiebert,* The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom, *Free Press, 1999.*

very influential on state departments of education, school districts, grant agencies, and textbook publishers. These organizations appreciated the fact that a knowledgeable group had organized the subject and laid out general recommendations. In the absence of any federal standards, it became virtually mandatory for textbook publishers to claim that their books adhered to the NCTM standards, despite the fact that some of the books were the same old thing with cosmetic changes. Many (probably not most) teachers began to rethink their teaching methods, and many districts tried new curricula.

"Math wars" and backlashes notwithstanding, I expect the NCTM standards will continue to serve as national guidelines. Since we may never have a national curriculum, this is the closest we will get to national standards. The NCTM is a respected professional organization, and it has done a careful job on the *Standards*.

The updated version of the *Standards*, released this year under the title *Principles and Standards for School Mathematics*, was written with significant input from mathematicians; this was not the case with the 1989 *Standards*. Stung by misunderstandings of recommendations such as "decreased attention to problems by type" and accusations of removing calculation and proof from the curriculum, the NCTM has bent over backwards to be clear about its recommendations, supporting them with explicit research citations, sample problems, vignettes of lessons, and samples of student work.

Principles and Standards is intended to be read by many different groups: mathematics teachers and supervisors, local and state educational administrators, mathematicians, politicians, parents, and business and community leaders.

The document is refreshingly free of mathematics education jargon. The writing is nonpedantic and readable by any interested, educated person. Readers will find little gems of mathematics lessons, such as the young child who discovers, using a calculator, that one cannot ever get to 100 by counting by 3s, then uses other mathematical props to explain why.

The graphic design is clean and restful to the eye. Unlike in many K–12 mathematics textbooks, there are no extraneous elements. In addition to text there are mathematical illustrations, diagrams and graphs, reproductions of student work, references to "E-examples" (interactive activities or video clips on the *Standards* Web site), and photographs of classrooms showing ways of organizing a mathematics class other than by having students sit in rows of desks, listening or writing.

Principles and Standards is carefully organized: the edges of pages are color-coded by grade band, with the ten standards listed down the side as tabs. So it is easy to find, say, "Geometry" in grades 9–12, or "Reasoning and Proof" in pre-K–2. The same material and more is on the Web; eventually the Web version will include a search engine, more E-examples, video clips, and other supporting material. The Web version is especially convenient for novices in mathematics education: one does not have to invest the time or money to order a paper copy.

Chapter 8, "Working Together to Achieve the Vision", outlines what must be done and by whom to successfully reform mathematics education. Much of the vitriol in the "math wars" that broke out in the 1990s comes from deeply held beliefs about mathematics education: that traditional Euclidean geometry is the best, or the worst, way to teach mathematical reasoning; that paper-and-pencil calculation is the key to, or a major impediment to, understanding numbers; that calculators should not be used until paper-and-pencil arithmetic has been mastered or that they should replace paper-and-pencil arithmetic. Mathematics education research does not give definitive answers to these questions, and many thoughtful teachers would agree with parts of each opinion. We need to set aside our differences and get down to the urgent business of convincing the American citizenry that teachers and schools need respect and financial support, convincing administrators that teachers need continuing professional development, convincing current and future teachers that knowing more mathematics is a vital professional requirement, and convincing students that it is possible and important to understand mathematics.

Reading this careful document from the NCTM is a good start.

How To Obtain *Principles and Standards*

Principles and Standards for School Mathematics is available on the World Wide Web, on paper, and on CD-ROM.

The Web version is available at `http://standards.nctm.org/`. A PDF version may be purchased online at `http://www.nctm.org/standards/`. Paper and CD-ROM versions may be ordered on the Web at `http://www.nctm.org/standards/buyonline/` or by telephone at 800-253-7566 (from overseas, 1-703-620-9840, extension 2601). The postal address for orders is:

National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 20191-9988

The book (#719) is $45 (NCTM members $36), the CD-ROM (#736) is $30 (members $24), and the PDF version is $30 (members $24). Those purchasing paper copies of the document can also request a free CD-ROM. For orders under $50 there is a shipping and handling charge of $7. Discounts are available on larger orders; consult the NCTM for further details.

—*A. J.*

Herbert Clemens

Principles and Standards for School Mathematics (PSSM) seems pretty good to me! Who can argue with a document that sets forth, in a very clear and well-organized manner, the consensus agenda for the subject matter to be learned in the K–12 cycle? Who can argue with the carefully prepared and documented classroom vignettes to illustrate student thinking and activity? Certainly no one with much classroom experience.

On the other hand, who can *not* argue with the suggestions for the teaching of the subject matter and the tentative indications of mathematics education research included in PSSM to bolster those suggestions? And that argument is precisely the point. There is no consensus on teaching mathematics, and perhaps in the best of all possible worlds, there need not be one. In any case, each individual teacher and user of PSSM will have to critique, accept, reject, incorporate, and adapt what is put forth in the document as it pertains to how teachers do their jobs. So if there is a danger related to PSSM, it does not seem to me to be a danger in the document itself. The danger is that the individual user, for whatever reason, cannot or will not read the document critically; the danger is greatest with regard to teaching methods. A teacher can read it and "check off" mathematical topics, but cannot read it and "check off" teaching methods, even though I firmly believe that there are many teachers and mathematics supervisors around the country who will do just that.

In fact, PSSM sets a very high standard for the individualization of teaching and learning, perhaps a higher standard than the U.S. market will bear at this point in history. The last time the bar was raised too high, the result was the so-called "new math", which cannot be judged a success by any measure. PSSM reflects much that is good in our attempts to rethink and to yet again reform mathematics education. But in spite of all that has been learned and in spite of the protestations of the finest researchers and teachers, it remains a fair question to ask whether PSSM falls prey to the same fundamental mistake of raising the bar too high. It is not a question of what PSSM says, but of how it will be read by teachers and administrators.

Which brings me to PSSM and the "math wars":

> …most of the arithmetic and algebraic procedures long viewed as the heart of the school mathematics curriculum can now be performed with hand-held calculators. Thus, more attention can be given to understanding the number concepts and the modeling of procedures used in solving problems. (PSSM, page 20)

The "math wars" that have flared up yet again in and outside our profession are encapsulated in the word "thus" in the above quotation from PSSM.

- *Traditionalists*: The above implication is a complete non sequitur, dangerous in the hands of all but the most mathematically sophisticated of school teachers.
- *Constructivists*: Learning procedures without conceptual understanding is pointless and ultimately useless. Anyway, machines can do procedures far better than humans.

What is perhaps underappreciated in such debates is the fact that rote learning is not the enemy of mathematical understanding, but an essential vehicle to it. For example, many if not most students cannot develop an intuition for abstraction and symbolic manipulation without first correctly manipulating a lot of symbols. Perhaps one of the principal disappointments in the "math wars" has been the failure of the participants to acknowledge that appreciation of this fact is shared by both sides.

PSSM begins to recognize the synergy between rote training and conceptual learning. But, as is natural for a document prepared and vetted by a large cross-section of the professional community, it reflects the fact that our community has a long way to go to get the balance right. The evolution of the reading wars finally led to the recognition of the necessary balance and constructive tension between "phonics" and "whole language". It is the same issue in mathematics.

Of course we all believe that teachers should teach with conceptual insight and clarity and should use all the tools at their disposal to convey the mathematical essence clearly and persuasively to as many students as possible. But let us not forget that training "by rote" is a less dangerous weapon in the hands of a teacher of limited mathematical preparation and understanding than are attempts to foster understanding in others that one does not have oneself.

> Proof is a very difficult area for undergraduate mathematics students. Perhaps students at the secondary level find proof so difficult because their only experience with writing proofs has been in a high school geometry course, so they have a limited perspective…. Reasoning and proof should be a consistent part of students' mathematical experience in prekindergarten through grade 12. Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts. (PSSM, page 56)

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If mathematical reasoning and proof are so difficult for undergraduate mathematics majors, is the same not true one hundred times over for most elementary and middle school teachers? And if that is the case, how on earth can we ask teachers to foster these skills in their students, except perhaps with vehicles (not yet invented) that work in a "teacher-proof" context? Teachers by and large are "kid people", not "math people". The "math people" are busy at an easier job, earning four times as much at some dot-com enterprise down the street. Let us be very careful about asking teachers for more than we have any right to expect. We run the risk of disrespecting the profession and its high level of skill and dedication and of foisting upon it the responsibility for remedying a deficiency that we as a society cause by the reward structure we have designed and perpetrated. It is of course essential that a document like PSSM lay out a vision and goals. But it is also important to guide each teacher of mathematics to find where he or she as an individual fits into the enterprise and give contexts for relatively successful teaching that fit an individual teacher's own mathematical knowledge and ability. On this point, perhaps PSSM still reflects a bit of a "one-size-fits-all" mentality. In fairness, however, there are indications throughout PSSM that show that we are beginning to understand and grapple with this problem.

Of course, when it comes to my own area, geometry, I must admit that I cringed when I read:

> Geometric ideas are useful in representing and solving problems in other areas of mathematics and in real-world situations, so geometry should be integrated when possible with other areas. (PSSM, page 41)

It seems to me that the integration of geometry with other areas (indeed, the disappearance of it as a separate course in some schools) is quite naturally the enemy of logical development. It is really hard to track a concept and its logical development when it pops up periodically only in service to some other part of mathematics or application. To me it would be far better for all concerned if high schools did a lot less in the way of advanced placement calculus courses and spent more time on deepening understanding of "elementary" mathematics like geometry and algebra/numerics. This change would be especially beneficial for "mathematically gifted" students, another group whose needs PSSM begins to recognize.

All in all, I would like to take my hat off to the writers, reviewers, and others involved in the preparation of PSSM. It seems to me to be a responsible, insightful, balanced, and inspiring document. The problems of teaching and learning mathematics it addresses are fundamental ones fraught with minefields of competing ideologies and interests, yet the framework it lays out is mature, insightful, detailed, and even-handed. I hope only that those who read and use PSSM do so with as much thoroughness, responsibility, thoughtfulness, and critical spirit as the writers brought to their task.

Roger Howe

I would encourage AMS members who read the PSSM to keep in mind the larger picture. Elements of the larger picture include:

a) Here is a document that says that it is the ideas in mathematics that count. Four of the ten standards are about ideas—problem solving, reasoning and proof, communication, and connections.

I would be happy for the U.S. to commit to mathematics education with a heavy emphasis on ideas, especially since

b) PSSM explicitly corrects the lesson taken by many from the 1989 *Standards* that ability actually to do arithmetic does not matter. PSSM states clearly that skill at computation is a goal. In pre-K–2, students should learn "addition facts"—the sums of all pairs of single-digit numbers. In 3–5 they should also learn the multiplication tables and "develop fluency" with whole-number arithmetic—all four operations. In 6–8 they should "develop and analyze algorithms for computing with fractions, decimals and integers, and develop fluency in their use." There is even a statement on page 153 that students should know the multiplication tables by the end of fourth grade.

This does not mean that PSSM is a perfect document. A mathematically attentive reader will find items that jar at all levels and in most parts of the document, at least the ones that I looked at. Here are a few samples. On page 144, lines 4–7, the indicated calculation appears to produce a fifth-grader 2 meters tall and an adult with a height of 2.5 meters. On a higher level, on page 314, it is explained how to set up a coordinate system to produce a manageable proof that the medians of a triangle are concurrent. However, the "best", or mathematically most natural, proof of this is coordinate-free, with vectors, which are one of the recommended techniques, along with coordinates, in the first bullet of the third geometry expectation (page 308). Or, just across on page 315, why is so much effort expended with no attempt to formulate the precise relation between a transformation and a matrix: that the k-th column of the matrix is the image of the k-th standard basis

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vector? On a still higher level, a main expectation of the “Geometry” standard for 9–12 is the use of transformations and symmetry to analyze and explore mathematical situations. However, on page 300 in the “Algebra” standard, there is a discussion of “exponential functions” that is summarized by this sentence:

> This type of exploration should help students see that all functions of the form $f(x) = ab^x + c$ share certain properties.

Although changing the sign of a and changing b to $1/b$ are mentioned in the discussion, there is no hint that any function in this family can be transformed to any other by simple transformations of the coordinates (e.g., of the type $x \to \alpha x + \beta$, and similarly for y). This would seem to violate the “Connections” standard.

I believe that it would be a mistake to conclude from items like these that PSSM is a document that should be discounted. Such items are probably nearly unavoidable given the way it was created: by a committee of twenty-seven, chosen to represent a diversity of points of view on mathematics education. Most of its readers will not notice such things.

Another aspect of the larger picture is that PSSM is primarily concerned with curriculum. Curriculum is the aspect of mathematics education on which mathematicians tend to focus. However, I believe that curriculum is a secondary issue in U.S. mathematics education today. The primary issue is the capacity of the teaching corps. A superb curriculum is beside the point if it makes demands that a teacher cannot meet. A potentially superior curriculum may in practice be inferior if implemented incorrectly. Conversely, an excellent teacher can make up for a lot of deficiencies in a textbook or program. I feel that the main question facing U.S. mathematics education is how to create a teaching corps that can deliver instruction that will promote understanding, reasoning, connections, and communication in mathematics, as well as technical skill and broad knowledge. A main finding of mathematics education research during the past decade is that doing even elementary mathematics right involves substantial mathematical judgment and understanding, as well as an ability to listen to students and adapt instruction to their needs. We do not currently know how we can reliably produce teachers with this combination of skills. It seems clear that it will take sustained and purposeful effort. I would guess that it would need at least the following combination of ingredients.

1) More and better mathematical training of prospective teachers. This has several aspects:

a) More mathematics courses for primary (K–3) candidates.

b) Separate certification and substantially more training for the intermediate level (4–8).

c) Specially designed courses built to connect important mathematical ideas to classroom experience.

d) Particularly for elementary teacher candidates, instruction adapted to their state of preparedness.

2) Sustained in-service training focused on mathematics, including night and summer courses and teacher institutes.

3) Continued learning on the job, by self-study and collegial interaction.

Point (3) will probably require considerable support, at least at first. The in-service training of point (2) could provide some of this support. Support could also come from mathematics resource teachers or mathematics specialists in elementary schools.

Developing a system with these features requires mutually reinforcing action by educational policymakers, schools of education, and mathematics departments. The essential enabling role will fall to educational policymakers. To make the programs of study in points (1a) and (1b) viable, certification requirements will have to be beefed up, especially at the middle school level. Probably also there would need to be some fairly probing examination for certification. Time will have to be provided in the teacher’s work week for the activities of point (3), and incentives to ensure participation in activities of point (2) will likely also be needed. Given the appropriate certification policies and working conditions, education schools and mathematics departments would then have to cooperate to create the courses of point (1) and the programs of point (2), and these efforts would also require appropriate support. Steps in this direction, in Connecticut since the late 1980s and recently in Puerto Rico, have had encouraging results. Rather than debate at great length about curriculum, let us get serious about enabling teachers to develop the mathematical skills they need to teach well.

Mark Saul

Normally, one must not pass on information offered anonymously, but the present situation is something of a special case.

A colleague at a large state university discovered the manuscript transcribed below in a container that might be described as a metal shoe box. It seems to be a message from the future. Is it this, or a fraternity hoax? I offer this decision to the

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reader, together with the very real issues the text raises.

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A Brief Communication to the History Division of the American Mathematical Society

This communication describes a manuscript recently uncovered in the archives of the Lappan Memorial Library. As you know, the Great Comet of 2065 wiped out all electronic records, ushering in a Little Dark Age, from which we are just now recovering. The present manuscript represents a rare glimpse into a world over which history has drawn a curtain.

The manuscript is entitled *Principles and Standards for School Mathematics* (PSSM) and is a product of the National Council of Teachers of Mathematics (NCTM), an organization that of course still exists. The document begins with a "Statement of Intention":

> This document is intended to—
>
> • set forth a comprehensive and coherent set of goals for mathematics for all students… ;
>
> • serve as a resource for teachers, education leaders, and policy-makers… ;
>
> • guide the development of curriculum frameworks, assessments, and instructional materials;
>
> • stimulate ideas and ongoing conversations…. (PSSM, page 6)

This primary source offers a wealth of information about the era that produced it, too much to treat thoroughly in the present brief communication. We will therefore focus on one aspect: the role played by mathematics education in American society of the time and particularly the role played by mathematicians in education.

The document first lays out several basic tenets that we now take for granted. These are contained in a set of six overarching "principles" about the learning and teaching of mathematics, many of which sound dated. For example, the "Equity Principle":

> The vision of equity in mathematics education challenges a pervasive societal belief in North America that only some students are capable of learning mathematics…. Equity does not mean that every student should receive identical instruction; instead it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students…. Achieving equity requires a significant allocation of human and material resources…(PSSM, pages 12–14).

Why did the authors feel impelled to include such obvious and commonplace assertions? An historical reading will give us the answer. At the turn of the millennium, the United States found itself the strongest and wealthiest country in the world and in the history of the world. Yet its wealth was distributed with significant inequalities. No balance had yet been struck between the economic incentive necessary for productivity and a broad inclusion of the entire population in that productivity to protect it from the ravages of social divisions. This was the great historical task of the twenty-first century.

Other "principles" are likewise dated. The oddest thing, however, about this section of the document is that it treats almost nothing specific to mathematics. The principles stated mostly apply, with only slight adjustments, to the learning of any other subject.

Let us turn to the role of the mathematician in education at the time, a role that was not yet well defined. Some mathematicians involved themselves directly in teaching precollege students and even contributed towards an understanding of teaching and learning by doing educational research. Many more mathematicians were involved in preparing "preservice" teachers, a process that was then rather haphazard (the current document acknowledges but does not detail the problems in this area). Still others assumed the role of critic, commenting on the work of others without performing any themselves. No one of these roles is today thought inappropriate, but no one of them dominates any individual mathematician's activities as it did in the past.

This lack of balance between research and outreach was a grave fault in the structure of the American mathematical community. Even at that time there were models, such as those in Eastern Europe, where mathematicians were much more closely and successfully integrated into the educational efforts of the nation.

The main part of the document, the ten "Standards", tells us more about this problem. Some of the labels on these standards are indeed strange. Today, we would think of algebra as essentially the study of binary operations, so that a "Number and Operation" standard would grow, as the grade levels get higher, into a standard for algebra. But it turns out that "Numbers and Operations" discusses mostly the relationship between operations and algorithms, then a sore point in education. At the time, many people erroneously assumed that a mere mastery of algorithms would somehow induce an understanding of arithmetic. It seems that this special standard was needed to address this error.

Even with the burden of this historical baggage, the wording of this standard is strikingly modern. There are important attempts to use findings from the nascent field of cognitive psychology, especially for the early years. There are concrete checkpoints and benchmarks offered, both in understanding and in computational ability. There are fascinating ideas offered about the use of technology. It is interesting that even with the few resources developed at the time and the small bits of knowledge accumulated, the authors got right many things that have been confirmed over time. It seems that a collective intuition about education sometimes proves accurate, a lesson we might take in our own time.

The "Number and Operations" standard for grades 9–12 shows most clearly the need for the input of the mathematician. Much is made of the study of vectors and matrices and the exploitation of parallels in structure between these objects and various number systems. But the discussion contains significant gaps. The next generation of mathematicians was able to work more closely with educators and examined more closely the mathematical and cognitive structures involved here to find a pedagogy that reflects these structures.

The "Algebra" standard is not that at all, but a hodgepodge of mathematical ideas packaged and labeled for convenience. Throughout the grades the study of algebra is confounded with the study of functions, of models, of pattern, and of several other important mathematical concepts related to but not part of algebra. One might well imagine the lumping and splitting that resulted in the sometimes strange taxonomy of the PSSM. The authors' intuitions are mostly correct but have not been focused through the lens of the mathematician.

There remains in this document a trace of a tension that permeated the decade that preceded it. Many educators had decided, largely without evidence, that mathematics could be made more "interesting" or "exciting" through the use of what was termed "real-world applications". However, this document is fairly balanced and includes a good mix of elegant "pure" mathematics and more rough-and-tumble applications.

The standard called "Geometry" is in much better shape, perhaps because this branch of mathematics has the longest and most successful history of any as a classroom subject. Even today we can think of ourselves as just coming out from under the shadow of Euclid. Geometry in the schools was for centuries synonymous with axiomatics, and there remain today mathematicians "married" to Euclid. For example, some think that straightedge and compass are the only viable construction tools and Euclid's set of axioms the only way to describe geometry. Euclid's was a towering accomplishment which has already lasted more than two millennia. We can continue to honor this accomplishment while extending it and finding alternatives.

During the twentieth century the wisdom of separating the studies of geometry and axiomatics became clear. For example, until about the era of the PSSM, geometry in elementary school consisted largely of the learning of nomenclature. In middle school, geometry was used as a platform to do algebra, if it was used at all. Geometry was started in earnest only in high school, because it was felt necessary to go about that study axiomatically. The "Geometry" standard in PSSM reflects a sea change in this view of the subject, describing a unified approach to the subject from early childhood through high school.

This standard, too, betrays the flaws of the culture that produced it. For example, mention is made (pages 235, 313) of how the introduction of coordinates links algebra with geometry, but other useful links between these two branches of mathematics are not mentioned. Another connection with algebra is missed when the composition of geometric transformations is brought up (page 314). Oddly, no mention is made of the noncommutative nature of this operation or its relationship to matrix multiplication.

It seems important to note that the lack of such mathematical connections cannot be attributed to organizational problems. The document describes an elaborate system of "Association Review Groups"—people in mathematics, statistics, and other fields who offered input. There are also mathematicians listed among the authors of the document.

Thus it is not an organization at fault, but a culture. However, the culture seems to have been more resilient than this one document shows. The process of setting down a single set of goals brought to the attention of the mathematical community exactly the gaps that seem so obvious to us now. Thus the document contributed as much in its flaws as in its successes.

The historical record includes a number of artifacts of the controversy that surrounded mathematics education at the time. In some cases both educators and mathematicians tried to think of PSSM as a curriculum in itself. We now know that there are good reasons why curriculum should not be set on this high level, but on a lower level. A paper curriculum changes when it is implemented in ways that are dependent on such factors as the teachers implementing the curriculum, the students' social environment, or the form of assessment of the curriculum. Especially in a setting as diverse as American society, we have found it more effective to implement refined versions of documents such as PSSM rather than impose curriculum from a level of management that cannot react to a close observation of the intended audience.

Another bone of contention seems to have been the mathematical accuracy of the document. Even today there are mathematicians who will inspect documents such as this one for errors, like someone attending a concert only to hunt for wrong notes. Such flaws do not significantly affect the strength of the document. A curriculum is not a mathematical argument. The latter has the structure of a balloon: a single hole renders it useless. But a document guiding educational practice can hold up despite a few weak points or even downright errors.

There are, however, deeper flaws, where important mathematical questions go begging. For example, on page 292 we have: "…students can learn to appreciate vectors as a means of simultaneously representing magnitude and direction." Does this really lie at the heart of the vector concept? Or on page 297: "Students should…study the behavior of polynomial, exponential, rational, and periodic functions." These categories are not parallel: perhaps students should also acquire a taste for Greek, Italian, Chinese, and orange food. These flaws are more important, and more worthy of the attention of the mathematician, than small slips in logic or exposition.

To conclude this communication I would like to offer the suggestion that this artifact from the past can teach us something about our own endeavors. As we see how our forebears struggled and erred, we must wonder what will be thought of our own efforts one hundred years hence. For we are not ready for The Definitive Document in education, and it is not clear that such a document is even possible. A historical perspective on this PSSM, as well as on our own work, will allow us to recognize our own errors and use them, as well as our triumphs, to guide our next steps.