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REPRESENTATION OF A COMPLETELY BOUNDED BIMODULE MAP

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ABSTRACT. In this paper, we give a representation for a completely bounded $A - B$ bimodule map into $B(H)$, where A and B are unital operator subalgebras of $B(H)$. When A and B are C^*-subalgebras we give a new proof of the Wittstock's theorem by using this representation. We also prove that a von Neumann algebra is an injective operator bimodule over its unital operator algebras if and only if it is a finitely injective operator bimodule.

1. INTRODUCTION

An operator space is a L^∞-matricially normed space (see [12]). A unital operator algebra is an operator space and is also a unital algebra with completely contractive multiplication (see [2]). An operator bimodule over two unital operator algebras is an operator space and is also a unital bimodule with completely contractive multiplication (see [3]). While there is an extensive literature on the representation of completely bounded and related types of linear maps (see [1, 3,7–10], and others), there has been relatively little done in the way of representating completely bounded bimodule maps. One notable exception is Smith's representation of completely bounded bimodule maps from $K(H)$ into $B(H)$. This paper shows in particular that M_6 is not an injective operator bimodule over a pair of unital operator subalgebras of M_6 (see [14]). We are motivated by this fact to study the representation of completely bounded bimodule maps and the injectivity of $B(H)$ as an operator bimodule.

In §2, we first give a representation for a completely bounded $A-B$ bimodule map into $B(H)$ when A and B are C^*-subalgebras of $B(H)$. Using this representation, we give a new proof of Wittstock's theorem. Later, we generalize the representation to the case that A and B are unital operator subalgebras of $B(H)$. In §3, we prove that a von Neumann algebra is an injective operator bimodule over two unital subalgebras if and only if it is a finitely injective operator bimodule.

Throughout this paper, all subspaces, operator subalgebras, operator subbimodules, etc., are closed. We use the term homomorphism for a bimodule

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map when no confusion may result. An embedding is an injective homomorphism. A homeomorphism is a surjective embedding. We do not distinguish between Y an operator subbimodule of X and a completely isometrical embedded copy of Y in X. Every vector space is over the complex numbers, and every map is linear.

Suppose X and Y are $A-B$ operator bimodules over unital operator algebras A and B. We denote by $\mathrm{Hom}(X, Y)$ the space of all completely bounded homomorphisms from X into Y. If X is a subset of a unital C^*-algebra, we denote by $C^*(X)$ the unital C^*-algebra generated by X.

2. Representation of a completely bounded bimodule map

We begin this section with a simple lemma (see [6]).

Lemma 2.1. *Suppose that A and B are operator algebras with 1_A and 1_B, respectively. Then an operator space X is an $A-B$ operator bimodule if and only if there exists a completely contractive trilinear map $\Phi: A \times X \times B \to X$ that satisfies*

$$\Phi(a_1a_2, x, b_1b_2) = \Phi(a_1, \Phi(a_2, x, b_1), b_2)$$

and

$$\Phi(1_A, x, 1_B) = x$$

for all $a_1, a_2 \in A$, $b_1, b_2 \in B$, and $x \in X$. Moreover, the multiplication is determined by Φ via the equation $\Phi(a, x, b) = axb$ for all $a \in A$, $b \in B$, and $x \in X$.

The following theorem gives us the representations of completely bounded C^*-bimodule maps.

Theorem 2.2. *Suppose that A and B are unital C^*-subalgebras of $B(H)$, where H is a Hilbert space. Suppose that X is an $A-B$ operator bimodule. Then every completely bounded $A-B$ bimodule map ϕ from X into $B(H)$ has a representation $(V_1, \pi_1, \theta, \pi_2, V_2, K)$, where π_1 and π_2 are $*$-representations of A and B on a Hilbert space K, θ is a complete contraction from X into $B(K)$, and $H \overset{V_2}{\to} K \overset{V_1}{\to} H$ are bridging maps such that*

$$\phi(x) = V_1\theta(x)V_2;$$
$$\theta(axb) = \pi_1(a)\theta(x)\pi_2(b);$$
$$aV_1 = V_1\pi_1(a), \qquad V_2b = \pi_2(b)V_2;$$
$$\|\phi\|_{\mathrm{cb}} = \|V_1\|\,\|V_2\|$$

for all $a \in A$, $x \in X$, and $b \in B$.

Proof. Suppose that $(\tilde{\pi}_1, \tilde{\theta}, \tilde{\pi}_2, \widetilde{K})$ is a representation of X in Corollary 3.3 of [3], i.e., $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are $*$-representations of A and B on a Hilbert space $\widetilde{K}$ and $\tilde{\theta}: X \to B(\widetilde{K})$ is a complete isometry such that

$$\tilde{\theta}(axb) = \tilde{\pi}_1(a)\tilde{\theta}(x)\tilde{\pi}_2(b)$$

for all $a \in A$, $x \in X$, and $b \in B$. Applying Lemma 2.1 above, we see that $\tilde{\theta}(X)$ is an $A-B$ operator bimodule with the bimodule multiplication given by $ayb = \tilde{\pi}_1(a)y\tilde{\pi}_2(b)$ for all $a \in A$, $Y \in \tilde{\theta}(X)$, and $b \in B$. Moreover, $\overline{\phi} = \phi \circ \tilde{\theta}^{-1}$ is a completely bounded $A-B$ bimodule map from $\tilde{\theta}(X)$ into

$B(H)$. Therefore, there exists a $*$-representation π of $B(\widetilde{K})$ on some Hilbert space K and bridging maps $H \overset{\widetilde{V}_2}{\to} K \overset{\widetilde{V}_1}{\to} H$ such that

$$\overline{\phi}(y) = \widetilde{V}_1 \pi(y) \widetilde{V}_2$$

for all $y \in \tilde{\theta}(X)$ and $\|\phi\|_{\mathrm{cb}} = \|\widetilde{V}_1\| \, \|\widetilde{V}_2\|$ (see [7]). Since for each $x \in X$,

$$\phi(x) = \overline{\phi}(\theta(x)) = \widetilde{V}_1 \pi(\tilde{\pi}_1(\mathbf{1}))\pi(\tilde{\theta}(x))\pi(\tilde{\pi}_2(\mathbf{1}))\widetilde{V}_2,$$

we may assume that $\widetilde{V}_1 = \widetilde{V}_1 \pi(\tilde{\pi}_1(\mathbf{1}))$, $\widetilde{V}_2 = \pi(\tilde{\pi}_2(\mathbf{1}))\widetilde{V}_2$, where $\mathbf{1}$ is the unit of $B(H)$.

Let $P\colon K \to [\pi(\tilde{\theta}(X))\widetilde{V}_2 H]$ be the orthogonal projection onto $[\pi(\tilde{\theta}(X))\widetilde{V}_2 H]$. Then

$$\overline{\phi} = \widetilde{V}_1 \pi \widetilde{V}_2 = \widetilde{V}_1 P \pi \widetilde{V}_2 .$$

Since $\pi(\tilde{\pi}_1(a))\pi(\tilde{\theta}(x)) = \pi(\tilde{\theta}(ax))$ for all $a \in A$ and $x \in X$, we have $P \in \pi(\tilde{\pi}_1(A))'$, the commutant of $\pi(\tilde{\pi}_1(A))$. Moreover, since for each $a \in A$ and $x \in X$,

$$\widetilde{V}_1 \pi(\tilde{\pi}_1(a))\pi(\tilde{\theta}(x))\widetilde{V}_2 = a\widetilde{V}_1 \pi(\tilde{\theta}(x))\widetilde{V}_2,$$

we have

$$\widetilde{V}_1 \pi(\tilde{\pi}_1(a))P = a\widetilde{V}_1 P$$

for all $a \in A$. Let $Q\colon K \to [\pi(\tilde{\theta}(X))^* P\widetilde{V}_1^* H]$ be the orthogonal projection onto $[\pi(\tilde{\theta}(X))^* P\widetilde{V}_1^* H]$. Since $P \in \pi(\tilde{\pi}_1(A))'$ and

$$\widetilde{V}_1 P \pi(\tilde{\theta}(x))\widetilde{V}_2 b = \widetilde{V}_1 P \pi(\tilde{\theta}(x))\pi(\tilde{\pi}_2(b))\widetilde{V}_2$$

for all $x \in X$ and $b \in B$, we have

$$b^* \widetilde{V}_2^* Q = \widetilde{V}_2^* \pi(\tilde{\pi}_2(b))^* Q$$

by taking adjoints. Thus,

$$Q\widetilde{V}_2 b = Q\pi(\tilde{\pi}_2(b))\widetilde{V}_2$$

for all $b \in B$. Since

$$\pi(\tilde{\pi}_2(b))^* \pi(\tilde{\theta}(x))^* P\widetilde{V}_1^* h = \pi(\tilde{\theta}(xb))^* P\widetilde{V}_1^* h$$

for all $b \in B$, $x \in X$, and $h \in H$, we have $Q \in \pi(\tilde{\pi}_2(B))'$. For any $x \in X$, h_1, $h_2 \in H$

$$\begin{aligned}(\phi(x)h_1, h_2) &= (\widetilde{V}_1 P \pi(\tilde{\theta}(x))\widetilde{V}_2 h_1, h_2) \\ &= (\widetilde{V}_2 h_1, \pi(\tilde{\theta}(x))^* P\widetilde{V}_1^* h_2) \\ &= (\widetilde{V}_2 h_1, Q\pi(\tilde{\theta}(x))^* P\widetilde{V}_1^* h_2) \\ &= (\widetilde{V}_1 P \pi(\tilde{\theta}(x))Q\widetilde{V}_2 h_1, h_2).\end{aligned}$$

Therefore,

$$\phi(x) = \overline{\phi}(\tilde{\theta}(x)) = \widetilde{V}_1 P \pi(\tilde{\theta}(x))Q\widetilde{V}_2 .$$

Now setting $V_1 = \widetilde{V}_1 P$, $V_2 = Q\widetilde{V}_2$, $\pi_1 = \pi \circ \tilde{\pi}_1$, $\pi_2 = \pi \circ \tilde{\pi}_2$, and $\theta = P(\pi \circ \tilde{\theta})Q$, we obtain the representation $(V_1, \pi_1, \theta, \pi_2, V_2, K)$ with the properties claimed in the theorem. □

Remark 2.1. The representation in Theorem 2.2 depends on the representation of the $A-B$ operator bimodule X. We will use this to give a new and totally different approach to the proof of Wittstock's theorem (cf. [15, 5]).

Suppose that A and B are unital operator algebras, and suppose that X is an $A-B$ operator bimodule. Recall that X is an injective $A-B$ operator bimodule if for each $A-B$ operator submimodule Y_1 of an $A-B$ operator bimodule Y and each completely bounded homomorphism $\phi: Y_1 \to X$ there exists a completely bounded homomorphism $\tilde{\phi}: Y \to X$ which extends ϕ and has the same cb-norm. In other words, X is an injective object in the category of $A-B$ operator bimodules and completely bounded homomorphisms (see [5]).

Theorem 2.3. *Suppose that A and B are unital C^*-subalgebras of $B(H)$, where H is a Hilbert space. Then $B(H)$ is an injective $A-B$ operator bimodule.*

Proof. Suppose that X is an $A-B$ operator subbimodule of an $A-B$ operator bimodule Y. Suppose $\phi \in \mathrm{Hom}(X, B(H))$. Suppose that $(\tilde{\pi}_1, \tilde{\theta}, \tilde{\pi}_2, \widetilde{K})$ is a representation of Y. Then $(\tilde{\pi}_1, \tilde{\theta}|_X, \tilde{\pi}_2, \widetilde{K})$ is a representation of X. By using the notation in the proof of Theorem 2.2, ϕ has a representation $(V_1, \pi_1, \hat{\theta}, \pi_2, V_2, K)$ with the properties described there, where $\hat{\theta} = P(\pi \circ \tilde{\theta}|_X)Q$. Now if we replace $\hat{\theta}$ by $\theta = P(\pi \circ \theta)Q$, then it is easy to see that $(V_1, \pi_1, \theta|_X, \pi_2, V_2, K)$ is a representation of ϕ with the properties claimed in Theorem 2.2. Moreover,

$$\begin{aligned} \theta(ayb) &= P\pi(\tilde{\pi}_1(a))\pi(\tilde{\theta}(y))\pi(\tilde{\pi}_2(b))Q \\ &= \pi_1(a)\theta(y)\pi_2(b) \end{aligned}$$

for all $a \in A$, $y \in Y$, and $b \in B$. Let $\tilde{\phi}: Y \to B(H)$ be given by the representation $(V_1, \pi_1, \theta, \pi_2, V_2, K)$; i.e., let $\tilde{\phi} = V_1\theta V_2$. Then $\tilde{\phi} \in \mathrm{Hom}(Y, B(H))$, extends ϕ, and has the same cb-norm $\|\phi\|_{\mathrm{cb}}$. □

When A and B are unital operator algebras, we still have the same form representation for a completely bounded $A-B$ bimodule map as we do in the case $A-B$ are unital C^*-algebras. However, the representation tells less information than it does in the latter case.

Corollary 2.4. *Suppose that A and B are unital operator algebras of $B(H)$, where H is Hilbert space. Suppose that X is an $A-B$ operator bimodule. Then every completely bounded $A-B$ bimodule map ϕ from X into $B(H)$ has representation $(V_1, \pi_1, \theta, \pi_2, V_2, K)$, where π_1 and π_2 are $*$-representation of $C^*(A)$ and $C^*(B)$ on a Hilbert space K, θ is a complete contraction from X into $B(K)$, and $H \xrightarrow{V_2} K \xrightarrow{V_1} H$ are bridging maps such that*

$$\begin{gathered} \phi(x) = V_1\theta(x)V_2; \\ \theta(axb) = \pi_1(a)\theta(x)\pi_2(b); \\ aV_1 = V_1\pi_1(a), \qquad V_2 b = \pi_2(b)V_2; \\ \|\phi\|_{\mathrm{cb}} = \|V_1\|\,\|V_2\| \end{gathered}$$

for all $a \in A$, $x \in X$, and $b \in B$.

Proof. By a theorem in [6], there exists a completely bounded $C^*(A)-C^*(B)$ bimodule map $\tilde{\phi}: \widetilde{X} \to B(H)$ such that $\phi = \tilde{\phi} \circ \alpha$ and $\|\phi\|_{\mathrm{cb}} = \|\tilde{\phi}\|_{\mathrm{cb}}$, where $\widetilde{X}$

is a dilation of X which is a $C^*(A)-C^*(B)$ operator bimodule and $\alpha: X \to \widetilde{X}$ is a complete contractive $A-B$ bimodule map. Applying Theorem 2.2 to $\tilde{\phi}$ and then restricting to X, we get the representation for ϕ. □

Remark 2.2. It is easy to see that the representation in Corollary 2.4 depends on the dilation $\widetilde{X}$ of X. We may not use Corollary 2.4 to get an analogous result of Theorem 2.3 when A and B are unital operator algebras. The reason is that when X is an $A-B$ operator sub-bimodule of an $A-B$ operator bimodule Y, the dilation $\widetilde{X}$ is not necessarily a $C^*(A)-C^*(B)$ operator subbimodule of the dilation $\widetilde{Y}$. In fact, M_6 is not an $A-B$ operator bimodule for some unital operator subalgebras A and B of M_6 (see [14]). The following section will give a sufficient and necessary condition for $B(H)$ to be an injective $A-B$ operator bimodule for unital operator subalgebras A and B of $B(H)$.

3. Injectivity of operator bimodules

We say that an $A-B$ operator bimodule is finitely generated if there exists a finite subset F of X such that $X = [AFB]$. The concept defined in the following definition seems to be a weaker notion than injectivity.

Definition 3.1. An $A-B$ operator bimodule X is called a finitely injective $A-B$ operator bimodule if for any two finitely generated $A-B$ operator bimodules X_1 and X_2 where X_1 is an $A-B$ operator subbimodule of X_2 and any $\phi \in \operatorname{Hom}(X_1, X)$ there is a $\tilde{\phi} \in \operatorname{Hom}(X_2, X)$ which extends ϕ and has the same cb-norm. Roughly speaking, X is an injective object in the category of finitely generated $A-B$ operator bimodules and completely bounded homomorphism.

The following theorem shows that injectivity and finite injectivity of operator bimodules are the same for von Neumann algebras. It should provide a useful tool to deal with the injectivity question for operator bimodules.

Theorem 3.1. *Suppose that $\mathscr{D}$ is a von Neumann algebra. Suppose that A and B are unital operator subalgebras of $\mathscr{D}$. Then $\mathscr{D}$ is an injective $A-B$ operator bimodule if and only if $\mathscr{D}$ is a finitely injective $A-B$ operator bimodule.*

Proof. It is obvious that injectivity implies finite injectivity. Suppose $\mathscr{D}$ is a finitely injective $A-B$ operator bimodule. We prove $\mathscr{D}$ is an injective $A-B$ operator bimodule. Suppose that X_1 is an $A-B$ operator subbimodule of an $A-B$ operator bimodule X_2 and $\phi \in \operatorname{Hom}(X_1, \mathscr{D})$. Without loss of generality, we may assume that $\|\phi\|_{\text{cb}} = 1$. We claim that for each $x_0 \in X_2 \backslash X_1$ there is a $\phi_{x_0} \in \operatorname{Hom}([X_1 + [Ax_0B]], \mathscr{D})$ which extends ϕ with the same cb-norm.

In fact, we may assume that $\|x_0\| = 1$. Let $\mathscr{F}$ be the family of finite subset of X_1. Then $\mathscr{F}$ is a partial ordered space with the usual set-theoretic inclusion partial order. For each $F \in \mathscr{F}$, $\phi|_{[AFB]} \in \operatorname{Hom}([AFB], \mathscr{D})$. By the finite injectivity of $\mathscr{D}$, there is an extension $\phi_{x_0, F} \in \operatorname{Hom}([A(F \cup \{x_0\})B], \mathscr{D})$ of $\phi|_{[AFB]} \in \operatorname{Hom}([AFB], \mathscr{D})$ such that $\|\phi_{x_0, F}\|_{\text{cb}} = \|\phi|_{[AFB]}\|_{\text{cb}}$. For each $F \in \mathscr{F}$, there is a subset F_{x_0} of $\mathscr{D}$ consisting of all $y \in \mathscr{D}$ such that there is a $\psi \in \operatorname{Hom}([A(F \cup \{x_0\})], \mathscr{D})$ which extends $\phi|_{[AFB]}$ with the cb-norm less than or equal to 1 and such that $\psi(x_0) = y$. Then F_{x_0} is a nonempty closed subset of the closed unit ball, ball$(\mathscr{D})$, of $\mathscr{D}$ which is compact in the weak operator topology. In fact, by the above argument, $F_{x_0} \neq \varnothing$ and $F_{x_0} \subseteq \text{ball}(\mathscr{D})$ because $\|\psi\|_{\text{cb}} \leq 1$ and $\|x_0\| = 1$. Suppose that (y_λ) is a net in F_{x_0} the converges to some y in the weak operator topology. Since the ball($\mathscr{D}$) is compact in the weak operator topology, $y \in \text{ball}(\mathscr{D})$. Let $\phi_\lambda \in \text{Hom}([A(F \cup \{x_0\}B], \mathscr{D})$ be the extension of $\phi|_{[AFB]}$ such that $\phi_\lambda(x_0) = y_\lambda$ and $\|\phi_\lambda\| \leq 1$. Since $\phi_\lambda|_{[AFB]+Ax_0B}$ is totally determined by y_λ, the limit $\psi = W\text{-}\lim \phi_\lambda|_{[AFB]+Ax_0B}$ exists. Since the cb-norm is lower semicontinuous in the weak operator topology, we have $\psi \in \text{Hom}([AFB]+Ax_0B, \mathscr{D})$ and $\|\psi\|_{\text{cb}} \leq 1$. Since $[AFB]+Ax_0B$ is norm dense in $[A(F \cup \{x_0\})B]$, ψ may be uniquely continuously extended to $[A(F \cup \{x_0\})B]$. Denoting the extension by ψ also, we have $\psi \in \text{Hom}([A(F \cup \{x_0\})B], \mathscr{D})$, $\|\psi\|_{\text{cb}} \leq 1$, and $\psi(x_0) = y$. Therefore, $y \in F_{x_0}$ and F_{x_0} is closed in weak operator topology.

If $\{F_{i,x_0}, F_i \in \mathscr{F}, 1 \leq i \leq n\}$, $n \in \mathbf{N}$, is a finite subcollection of $\{F_{x_0}, F \in \mathscr{F}\}$, then $\bigcup F_i \in \mathscr{F}$ and $(\bigcup F_i)_{x_0} \subseteq F_{i,x_0}$ for all $1 \leq i \leq n$. Therefore, $\{F_{x_0}, F \in \mathscr{F}\}$ has finite intersection property. Since ball($\mathscr{D}$) is compact in the weak operator topology, there is a $y_0 \in \bigcap\{F_{x_0}, F \in \mathscr{F}\}$. Define $\phi_{x_0}: X_1 + Ax_0B \to \mathscr{D}$ in the following way: for each $x \in X_1 + Ax_0B$, there is a $F \in \mathscr{F}$ such that $x \in [A(F \cup \{x_0\})B]$; let $\phi_{F,x_0} \in \text{Hom}([A(F \cup \{x_0\})B, \mathscr{D})$ be such that $\phi_{F,x_0}(x_0) = y_0$ and $\|\phi_{F,x_0}\| \leq 1$; and set $\phi_{x_0}(x) = \phi_{F,x_0}(x)$. That ϕ_{x_0} is a well-defined homomorphism that follows from the definition of y_0. Moreover, $\|\phi_{x_0}\|_{\text{cb}} = \|\phi\|_{\text{cb}}$ because $1 = \|\phi\|_{\text{cb}} = \sup \|\phi|_{[AFB]}\|_{\text{cb}}$. Since $X_1 + Ax_0B$ is dense in the $[X_1 + [Ax_0B]]$, we may continuously extend ϕ_{x_0} to $[X_1 + [Ax_0B]]$, obtaining $\phi_{x_0} \in \text{Hom}([X_1 + [Ax_0B]], \mathscr{D})$ which extends ϕ with the same cb-norm.

Let $\mathscr{G}$ be the family of pairs (ϕ_Y, Y), where Y is an $A - B$ operator sub-bimodule of X_2 containing X_1 and $\phi_Y \in \text{Hom}(Y, \mathscr{D})$ which extends ϕ with the same cb-norm. By the argument just given, $\mathscr{G}$ is a nontrivial family. We give $\mathscr{G}$ the partial order defined by $(\phi_{Y_1}, Y_1) \preceq (\phi_{Y_2}, Y_2)$ if Y_1 is an $A - B$ operator sub-bimodule of Y_2 and $\phi_{Y_2}|_{Y_1} = \phi_{Y_1}$. By Zorn's lemma, there is a maximal element (ϕ_{Y_0}, Y_0). From the initial step, we see that $Y_0 = X_2$. Letting $\tilde{\phi} = \phi_{Y_0}$ yields the desired extension. □

References

1. William Arvesion, *Subalgebras of C^*-algebra*, Acta Math. **123** (1969), 141–224.
2. David P. Blecher, Zhong-Jin Ruan, and Allian M. Sinclair, *A characterization of operator algebras*, J. Funct. Anal. **89** (1990), 188–175.
3. Erik Christensen, Edward G. Effros, and Allan Sinclair, *Completely bounded multilinear maps and C^*-algebras cohomology*, Invent. Math. **90** (1987), 279–296.
4. Erik Christensen and Allan Sinclair, *A survey of completely bounded operators*, Bull. London Math. Soc. **21** (1989), 417–448.
5. Paul S. Muhly and Qiyuan Na, *Extension of completely bounded $A - B$ bimodule maps*, preprint, 1992.
6. ——, *Dilation of operator bimodules*, preprint, 1992.
7. Vern I. Paulsen, *Completely bounded maps and dilations*, Pitman Res. Notes in Math. Ser., vol. 146, Longman, New York, 1986.
8. ——, *Every completely polynomially bounded operator is similar to a contraction*, J. Funct. Anal. **55** (1984), 1–17.
9. Vern I. Paulsen and R. R. Smith, *Multilinear maps and tensor norm on operator system*, J. Funct. Anal. **73** (1987), 258–276.
10. Vern I. Paulsen and Ching Yun Suen, *Commutant representation of completely bounded maps*, J. Operator Theory **131** (1985), 87–101.

11. A Guyan Robertson, *Injective matrical Hilbert spaces*, Math. Proc. Cambridge Philos. Soc. **110** (1991), 183–190.

12. Zhong-Jin Ruan, *Subspaces of C^*-algebras*, J. Funct. Anal. **76** (1988), 217–230.

13. ——, *Injectivity of operator spaces*, Trans. Amer. Math. Soc. **315** (1989), 89–104.

14. R. R Smith, *Completely bounded module maps and the Haagerup tensor product*, J. Funct. Anal. **102** (1991), 156–175.

15. G. Wittstock, *Extension of completely bounded C^*-module homomorphisms*, Proc. Conf. On Operator Algebras and Group Representations (Neptun, 1980), Pitman, New York, 1983, pp. 238–250.

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